Mark Scheme (Results)

Summer 2014

Pearson Edexcel GCE in Further Pure Mathematics FP2
(6668/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- M marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- B marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper
- ⎣ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Further Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q),$ where $|pq| = |c|$, leading to x = …

$(ax^2 + bx + c) = (mx + p)(nx + q),$ where $|pq| = |c|$ and $|mn| = |a|$, leading to x = …

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$, leading to x = …

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $(x^n \rightarrow x^{n-1})$

2. Integration

Power of at least one term increased by 1. $(x^n \rightarrow x^{n+1})$

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme		Marks
1.(a)	$\dfrac{2}{(r+2)(r+4)}=\dfrac{1}{r+2}-\dfrac{1}{r+4}$	Correct partial fractions. Can be seen in (b) – give B1 for that.	B1 (1)
(b)	$\displaystyle\sum_{r=1}^{n}\frac{2}{(r+2)(r+4)}=\sum_{r=1}^{n}\left(\frac{1}{r+2}-\frac{1}{r+4}\right)$		
	$=\dfrac{1}{3}-\dfrac{1}{5}+\dfrac{1}{4}-\dfrac{1}{6}+........$ $+\dfrac{1}{n+1}-\dfrac{1}{n+3}+\dfrac{1}{n+2}-\dfrac{1}{n+4}$	Attempts at least the first 2 terms and at least the last 2 terms as shown.(May be implied by later work) Must start at 1 and end at n	M1
	$=\dfrac{1}{3}+\dfrac{1}{4}-\dfrac{1}{n+3}-\dfrac{1}{n+4}$	M1: Identifies their four fractions that do not cancel. If all terms are positive this mark is lost. A1: Correct four fractions	M1A1
	$=\dfrac{7}{12}-\dfrac{1}{n+3}-\dfrac{1}{n+4}$		
	$=\dfrac{7(n+3)(n+4)-12(n+4)-12(n+3)}{12(n+3)(n+4)}$ $=\dfrac{7n^2+49n+84-12n-48-12n-36}{12(n+3)(n+4)}$	Attempt to combine at least 3 fractions, 2 of which have a function of n in the denominator and expands the numerator. As a minimim, the product of 2 linear factors must be expanded in the numerator.	M1
	$=\dfrac{n(7n+25)}{12(n+3)(n+4)}$ *	cso Must be factorised. If worked with r instead of n throughout, deduct last mark only.	A1 (5)
			Total 6
(b) Way 2	$\dfrac{7}{12}-\left(\dfrac{1}{n+3}+\dfrac{1}{n+4}\right)$		
	$=\dfrac{7}{12}-\left(\dfrac{n+4+n+3}{(n+3)(n+4)}\right)$		
	$=\dfrac{7(n+3)(n+4)-24n-84}{12(n+3)(n+4)}$		
	$=\dfrac{7n^2+49n+84-24n-84}{12(n+3)(n+4)}$	Attempt to combine at least 3 fractions, 2 of which have a function of n in the denominator and expands the numerator. Min as above	M1
	$=\dfrac{n(7n+25)}{12(n+3)(n+4)}$ *	cso	A1

Question Number	Scheme		Marks
2.	$\lvert 3x^2 - 19x + 20 \rvert < 2x + 2$		
	$3x^2 - 19x + 20 = 2x + 2$ $\Rightarrow 3x^2 - 21x + 18 = 0 \Rightarrow x = ..$	$3x^2 - 19x + 20 = 2x + 2$ and attempt to solve correctly May be solved as an inequality	M1
	$x = 1, \quad x = 6$	Both (ie critical values seen)	A1
	$-\left(3x^2 - 19x + 20\right) = 2x + 2$ $\Rightarrow 3x^2 - 17x + 22 = 0 \Rightarrow x = ..$	$-\left(3x^2 - 19x + 20\right) = 2x + 2$ and attempt to solve correctly May be solved as an inequality	M1
	$x = \frac{11}{3}, \quad x = 2$	Both (critical values seen) Accept awrt 3.67	A1
	$1 < x < 2, \quad \frac{11}{3} < x < 6$	Must be strict inequalities. Accept awrt 3.67 A1 either correct, A1 both correct. But give A1A0 if both correct apart from $\leqslant$ seen somewhere in the final answers. Give A1A0 if both correct and extra intervals seen	A1, A1
			(6)
			Total 6

If *no algebra* seen (implies a calculator solution) no marks.

With algebra:
M1 Squaring and reaching a quartic = 0
M1 Attempt to factorise and obtain at least one solution for x. Coefficient of x^4 and constant term correct for their quartic.
A1 Any 2 correct values
A1 All 4 correct values
Final 2 A marks as above

Accept set notation for the final 2 A marks. $x \in (1,2), \quad x \in \left(\frac{11}{3}, 6\right)$ not $[1,2]$

Question Number	Scheme		Marks
3.	$y=\sqrt{8+e^x}$		
	$y=\left(8+e^x\right)^{\frac{1}{2}} \Rightarrow \frac{dy}{dx}=\frac{1}{2}\left(8+e^x\right)^{-\frac{1}{2}}\times e^x$	M1: $\frac{dy}{dx}=k\left(8+e^x\right)^{-\frac{1}{2}}\times e^x$ A1: Correct differentiation	M1A1
	$\frac{d^2y}{dx^2}=\frac{1}{2}\left(8+e^x\right)^{-\frac{1}{2}}\times e^x-\frac{1}{4}\left(8+e^x\right)^{-\frac{3}{2}}\times e^{2x}$	M1: Correct use of the product rule $\frac{dy}{dx}=k\left(8+e^x\right)^{-\frac{1}{2}}\times e^x$ $\pm K\left(8+e^x\right)^{-\frac{3}{2}}\times e^{(2)x}$ A1: Correct second derivative with $e^x\times e^x$ or e^{2x}	M1A1
	$f(0)=3$	May only appear in the expansion	B1
	$f'(0)=\frac{1}{6},\ f''(0)=\frac{17}{108}$	Attempt both $f'(0)$ and $f''(0)$ with their derivatives found above	M1
	$(y=)3+\frac{1}{6}x+\frac{17}{216}x^2$	M1: Uses the correct Maclaurin series with their values. Accept 2 or 2! in x^2 term A1: Correct expression	M1 A1cso (8)
			Total 8
	Alternative Methods:		
	$e^x=1+x+\frac{x^2}{2!}+...$	2 or 2!	
	$y=\left(9+x+\frac{x^2}{2}...\right)^{\frac{1}{2}}$	M1: Subst corrrect expansion	M1
	$=3\left(1+\frac{x}{9}+\frac{x^2}{9\times 2}+...\right)^{\frac{1}{2}}$	B1: for 3 A1: for bracket	B1 A1
	$=3\left(1+\frac{1}{2}\left(\frac{x}{9}+\frac{x^2}{2\times 9}\right)+\frac{\frac{1}{2}\times\left(-\frac{1}{2}\right)}{2!}\left(\frac{x}{9}+\frac{x^2}{2\times 9}\right)^2\right)$	M1: Binomial expansion up to at least the squared term, 2 or 2! With squared term A1: Correct expansion ie contents of bracket correct	M1A1
	$=3+\frac{x}{6}+\frac{x^2}{12}-\frac{3}{8}\times\frac{x^2}{81}$	M1 Remove all brackets	M1
	$(y=)3+\frac{1}{6}x+\frac{17}{216}x^2$	M1: Combine x^2 terms and obtain a 3 term quadratic A1: Correct expression with or without $y=\ldots$	M1A1

By implicit differentiation: For the first 4 marks (rest as first method)

$$y^2=8+e^x$$

M1A1 $2y\frac{dy}{dx}=e^x$ M1A1 $2\frac{dy}{dx}+2y\frac{d^2y}{dx^2}=e^x$

Question Number	Scheme		Marks
4.(a)	$\cos 6\theta = \text{Re}[(\cos\theta + i\sin\theta)^6]$	Ignore any imaginary parts included in their expansion	
	$(\cos\theta + \text{i}\sin\theta)^6 = c^6 + 6c^5\text{i}s + 15c^4\text{i}^2s^2 + 20c^3\text{i}^3s^3 + 15c^2\text{i}^4s^4 + 6c\text{i}^5s^5 + \text{i}^6s^6$		M1
	Attempt to expand correctly or only show real terms (May be implied) Often seen with powers of i simplified. If $\text{i}s^n$ seen, but becomes $\text{i}^n s^n$ (oe) later, deduct the final A mark of (a) even if no further errors.		
	$\cos 6\theta = c^6 - 15c^4s^2 + 15c^2s^4 - s^6$	M1: Attempt to identify real parts. These 2 M marks may be awarded together A1: Correct expression	M1A1
	$= c^6 - 15c^4(1-c^2) + 15c^2(1-c^2)^2 - (1-c^2)^3$		M1
	Correct use of $s^2 = 1 - c^2$ in all their sine terms		
	$\cos 6\theta = c^6 - 15c^4 + 15c^6 + 15c^2(1 - 2c^2 + c^4) \; - (1 - 3c^2 + 3c^4 - c^6)$		
	$\cos 6\theta = 32\cos^6\theta - 48\cos^4\theta + 18\cos^2\theta - 1$ *	($\cos 6\theta$ must be seen somewhere)	A1cso
			(5)
(b)	$64\cos^6\theta - 96\cos^4\theta + 36\cos^2\theta - 3 = 0$ $\Rightarrow 2\cos 6\theta - 1 = 0 \therefore \underline{\cos 6\theta = \tfrac{1}{2} \text{ or } 0.5}$	M1: Uses part (a) to obtain an equation in $\cos 6\theta$ A1: Correct underlined equation	M1A1
	$\cos 6\theta = \tfrac{1}{2} \Rightarrow (6\theta =) \; \dfrac{\pi}{3}, \dfrac{5\pi}{3}, \dfrac{7\pi}{3}$		
	$\theta = \dfrac{\pi}{18}, \dfrac{5\pi}{18}, \dfrac{7\pi}{18}$	M1: Valid attempt to solve $\cos 6\theta = k,\ -1 \le k \le 1$ leading to $\theta = \ldots$ Can be degrees A1 2 correct answers A1 3rd correct answer with no extras within the range, ignore extras outside the range. Must be radians Answers in degrees or decimal answers score A0A0	M1 A1A1
			(5)
			Total 10

Question Number	Scheme		Marks
5.(a)	$\frac{d^2y}{dx^2}+2\frac{dy}{dx}+10y=27e^{-x}$		
	$m^2+2m+10\ (=0)\Rightarrow m=...$	Form and solve the aux equation	M1
	$m=-1\pm 3i$		A1
	$(y=)e^{-x}(A\cos 3x+B\sin 3x)$ or $(y=)Ae^{(-1+3i)x}+Be^{(-1-3i)x}$	$y=$ not needed May be seen with θ instead of x	A1
	$y=ke^{-x},\ y'=-ke^{-x},\ y''=ke^{-x}$	$y=ke^{-x}$ and attempt to differentiate twice	M1
	$e^{-x}(k-2k+10k)=27e^{-x}\Rightarrow k=3$		A1
	$y=e^{-x}(A\cos 3x+B\sin 3x+3)$ or $y=Ae^{(-1+3i)x}+Be^{(-1-3i)x}+3e^{-x}$	Must be x and have $y=$ ....Ignore any attempts to change the second form. (But see note at end about marking (b)) ft, so y = their CF + their PI	B1ft (NB A1 on e-PEN)
			(6)
(b)	$x=0,\ y=0\Rightarrow A=(-3)$	Uses $x=0,\ y=0$ in an attempt to find A	M1
	$y'=-e^{-x}(A\cos 3x+B\sin 3x+3)+e^{-x}(3B\cos 3x-3A\sin 3x)$	M1: Attempt to differentiate using the product rule, with A or their value of A A1: Correct derivative, with A or their value of A	M1A1
	$x=0,\ y'=0\Rightarrow B=0$	M1: Uses $x=0,\ y'=0$ and their value of A in an attempt to find B A1: $B=0$	M1A1
	$y=e^{-x}(3-3\cos 3x)$ oe	cao and cso	A1 (6)
			Total 12
	Alternative for (b) using	$y=Ae^{(-1+3i)x}+Be^{(-1-3i)x}+3e^{-x}$	
	$x=0,\ y=0$ to get an equation in A and B	May come from the real part of their derivative instead	M1
	$y'=(-1+3i)Ae^{(-1+3i)x}+(-1-3i)Be^{(-1-3i)x}-3e^{-x}$	M1: Attempt differentiation using chain rule A1: Correct differentiation	M1A1
	$x=0,\ y'=0\Rightarrow -A-B-3=0$ from real parts and $3A-3B=0$ from imaginary parts So $A=B=-\frac{3}{2}$	M1: Uses $x=0,\ y'=0$ and equates imaginary parts to obtain a second equation for A and B and attempts to solve their equations A1: $A=B=-\frac{3}{2}$	M1A1
	$y=-\frac{3}{2}e^{(-1+3i)x}-\frac{3}{2}e^{(-1-3i)x}+3e^{-x}$	A1: Ignore any attempts to change.	A1

Some may change the second form in (a) before proceeding to (b). If their changed form is correct, all marks for (b) are available; if their changed form is incorrect only M marks are available.

Question Number	Scheme		Marks
6.(a)	$w=\dfrac{4(1-i)z-8i}{2(i-1)z-i}$		
	Method 1………:Substituting $z=x+xi$	**at the start**	
	$w=\dfrac{4(1-i)(x+xi)-8i}{2(i-1)(x+xi)-i}$	Substitutes for z	M1
	$w=\dfrac{4(x+xi-xi+x)-8i}{2(xi-x-x-xi)-i}$	M1: Attempt to expand numerator and denominator A1: Correct expression	M1A1
	$\dfrac{8i-8x}{4x+i}\cdot\dfrac{4x-i}{4x-i}$	M1: Multiplies numerator and denominator by the conjugate of their denom. No expansion needed A1: Uses correct conjugate (not ft)	M1A1
	$=\dfrac{-32x^2+40xi+8}{16x^2+1}$	cso Award only if final answer is correct and follows correct working	B1
	NB: The B mark appears first on e-PEN but will be awarded last		
			(6)
	Method 2: if they proceed without $y=x$ (substitution may happen anywhere in the working)		
	$w=\dfrac{(1-i)z-8\text{i}}{2(-1+\text{i})-\text{i}}=\dfrac{4(1-\text{i})(x+\text{i}y)-8\text{i}}{2(-1+\text{i})(x+\text{i}y)-\text{i}}$	Substitutes for z	M1
	$=\dfrac{4(1-\text{i})x+4(1-\text{i})\text{i}y-8\text{i}}{2(-1+\text{i})x+2(-1+\text{i})\text{i}y-\text{i}}$	Attempt to expand numerator and denominator	M1
	$=\dfrac{4x+4y+(4y-4x-8)\text{i}}{-2x-2y+(2x-2y-1)\text{i}}$	Correct expression	A1
	$=\dfrac{4x+4y+(4y-4x-8)\text{i}}{-2x-2y+(2x-2y-1)\text{i}}\times\dfrac{-2x-2y-(2x-2y-1)\text{i}}{-2x-2y-(2x-2y-1)\text{i}}$ M1: Multiplies numerator and denominator by the conjugate of their denom. No expansion needed. A1: Uses correct conjugate. (not ft)		M1A1
	$=\dfrac{-16x^2-16y^2+12y-12x+8+(20x+20y)\text{i}}{8x^2+8y^2-4x+4y+1}$		
	$=\dfrac{-32x^2+40x\text{i}+8}{16x^2+1}$	cso Correct answer using $y=x$ Award only if final answer is correct and follows correct working	B1
	NB: The B mark appears first on e-PEN but will be awarded last		(6)

Question Number	Scheme		Marks
NB: The order of awarding the marks here has changed from the original mark scheme, but they must still be entered on e-PEN by their descriptors (M or A)			
6(b)	$u=\dfrac{-32x^2+8}{16x^2+1},\quad v=\dfrac{40x}{16x^2+1}$	Identifies u and v (Real and imaginary parts) May be implied by their working and may be in terms of x and y.	M1 1st M mark on e-PEN
	$\left(\dfrac{8-32x^2}{16x^2+1}-3\right)^2+\left(\dfrac{40x}{16x^2+1}\right)^2$	Substitutes for their u and v in the given equation. May be in terms of x and y. May have a, b, c instead of their values (which may be chosen by the candidate if unable to do (a))	dM1 2nd M mark on e-PEN
	$=\left(\dfrac{8-32x^2-48x^2-3}{16x^2+1}\right)^2+\left(\dfrac{40x}{16x^2+1}\right)^2$		
	$=\dfrac{\left(5-80x^2\right)^2}{\left(16x^2+1\right)^2}+\dfrac{1600x^2}{\left(16x^2+1\right)^2}$		
	$=\dfrac{6400x^4+800x^2+25}{\left(16x^2+1\right)^2}$	Combines to form a single correct fraction	A1 1st A mark on e-PEN
	$=\dfrac{25\left(16x^2+1\right)^2}{\left(16x^2+1\right)^2}=25$	$k=5$ or $k^2=25$ may (but need not) be seen explicitly	A1 2nd A mark on e-PEN
			(4)
			Total 10

Question Number	Scheme		Marks
	Way 1		
7.(a)	$v = y^{-3} \Rightarrow \frac{dv}{dy} = -3y^{-4}$	Correct derivative	B1
	$\frac{dy}{dx} = \frac{dy}{dv}\frac{dv}{dx} = -\frac{y^4}{3}\frac{dv}{dx}$ Or $-3y^{-4}\frac{dy}{dx}x - 3y^{-3} = -6x^4$	M1: Correct use of the chain rule A1: Correct equation	M1A1
	$-\frac{y^4}{3}\frac{dv}{dx}x + y = 2x^4y^4$		
	$-\frac{y^4}{3}\frac{dv}{dx}x + y = 2x^4y^4 \Rightarrow \frac{dv}{dx} - \frac{3v}{x} = -6x^3$	dM1: Substitutes to obtain an equation in v and x. A1: Correct completion with no errors seen	dM1A1
	Way 2		
	$y = v^{-\frac{1}{3}} \Rightarrow \frac{dy}{dv} = -\frac{1}{3}v^{-\frac{4}{3}}$	Correct derivative	B1
	$\frac{dy}{dx} = \frac{dy}{dv}\frac{dv}{dx} = -\frac{1}{3}v^{-\frac{4}{3}}\frac{dv}{dx}$	M1: Correct use of the chain rule A1: Correct equation	M1A1
	$-\frac{v^{-\frac{4}{3}}}{3}\frac{dv}{dx}x + v^{-\frac{1}{3}} = 2x^4v^{-\frac{4}{3}}$	dM1: Substitutes to obtain an equation in v and x.	dM1
	$-\frac{v^{-\frac{4}{3}}}{3}\frac{dv}{dx}x + v^{-\frac{1}{3}} = 2x^4v^{-\frac{4}{3}} \Rightarrow \frac{dv}{dx} - \frac{3v}{x} = -6x^3$	A1: Correct completion with no errors seen	A1
	Way 3 (Working in reverse)		
	$v = y^{-3} \Rightarrow \frac{dv}{dy} = -3y^{-4}$	B1: Correct derivative	B1
	$\frac{dv}{dx} = \frac{dv}{dy}\frac{dy}{dx} = -3y^{-4}\frac{dy}{dx}$	M1: Correct use of chain rule A1: Correct expression for dv/dx	M1A1
	$-3y^{-4}\frac{dy}{dx} - \frac{3y^{-3}}{x} = -6x^3$	M1: Substitutes correctly for $\frac{dv}{dx}$ and v in equation (II) to obtain a D.E. in terms of x and y only. A1: Correct completion to obtain equation (I) with no errors seen	dM1A1

Question Number	Scheme		Marks
7(b)	$I = e^{\int -\frac{3}{x} dx} = e^{-3\ln x} = \frac{1}{x^3}$	M1: $e^{\int \pm\frac{3}{x} dx}$ and attempt integration. If not correct, ln x must be seen. A1: $\frac{1}{x^3}$	M1A1
	$\frac{v}{x^3} = \int -6\,dx = -6x(+c)$	M1: $v \times$ their $I = \int -6x^3 \times$ their $I\,dx$ A1: Correct equation with or without + c	dM1A1
	$\frac{1}{y^3x^3} = -6x + c \Rightarrow y^3 = ...$	Include the constant, then substitute for y and attempt to rearrange to y^3 =.... or y = ... with the constant treated correctly	ddM1 dep on both M marks of (b)
	$y^3 = \frac{1}{cx^3 - 6x^4}$	Or equivalent	A1 (6) **Total 11**

Question Number	Scheme		Marks
	$r = 1+\tan\theta$		
8.(a)	$x = r\cos\theta \Rightarrow x = (1+\tan\theta)\cos\theta$	States or implies $x = r\cos\theta$	M1
	$x = \cos\theta + \sin\theta,\ \frac{dx}{d\theta} = \cos\theta - \sin\theta$	M1: Attempt to differentiate $x = r\cos\theta$ or $x = r\sin\theta$ A1: Correct derivative	M1A1
Alt for the 2 diff marks	$\frac{dx}{d\theta} = \sec^2\theta\cos\theta + (1+\tan\theta)(-\sin\theta)$	M1: Attempt to differentiate using product rule (dep on first M1) A1: correct (unsimplified) differentiation	
	$\frac{dx}{d\theta} = 0 \Rightarrow \tan\theta = 1 \Rightarrow \theta = ...$	Set their derivative = 0 and attempt to solve for θ (Dependent on second M mark above)	dM1
	$\theta = \frac{\pi}{4},\ r = 2$	Both	A1
	NB: Use of $x = r\sin\theta$ can score M0M1A0M1A0 max		(5)
(b)	$\int r^2 d\theta = \int (1+\tan\theta)^2 d\theta$	Use of $\int r^2 d\theta$ and $r = 1+\tan\theta$ No limits needed	M1
	$(1+\tan\theta)^2 = 1+2\tan\theta+\tan^2\theta$ $= 1+2\tan\theta+\sec^2\theta-1$	Expands and uses the correct identity	M1
	$\int (2\tan\theta+\sec^2\theta) d\theta$	Correct expression Need not be simplified, no limits needed.	A1
	$\left[2\ln\sec\theta+\tan\theta\right]_{\left(\frac{\pi}{4}\right)}^{\left(\frac{\pi}{3}\right)}$	M1: Attempt to integrate – at least one trig term integrated. Dependent on the second M mark A1: Correct integration. Need not be simplified or include limits.	dM1A1
	$R = \frac{1}{2}\left\{\left(2\ln\sec\frac{\pi}{3}+\tan\frac{\pi}{3}\right)-\left(2\ln\sec\frac{\pi}{4}+\tan\frac{\pi}{4}\right)\right\}$	Substitutes $\frac{\pi}{3}$ and their $\frac{\pi}{4}$ and subtracts (Dependent on 2 previous method marks in (b))	dM1
	$R = \frac{1}{2}\left\{\ln 2+\sqrt{3}-1\right\}$	Cao and cso	A1
			(7)
			Total 12

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with its registered office at Edinburgh Gate, Harlow, Essex CM20 2JE